

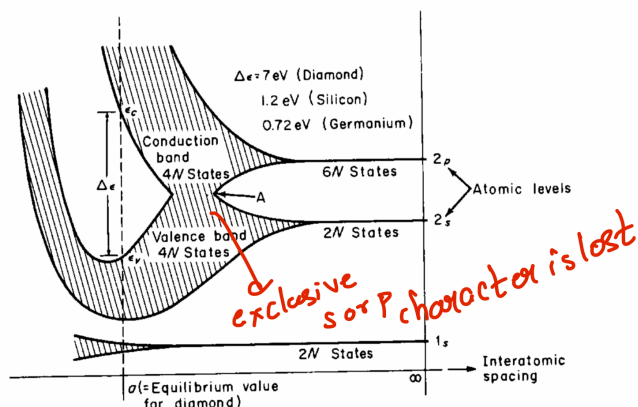
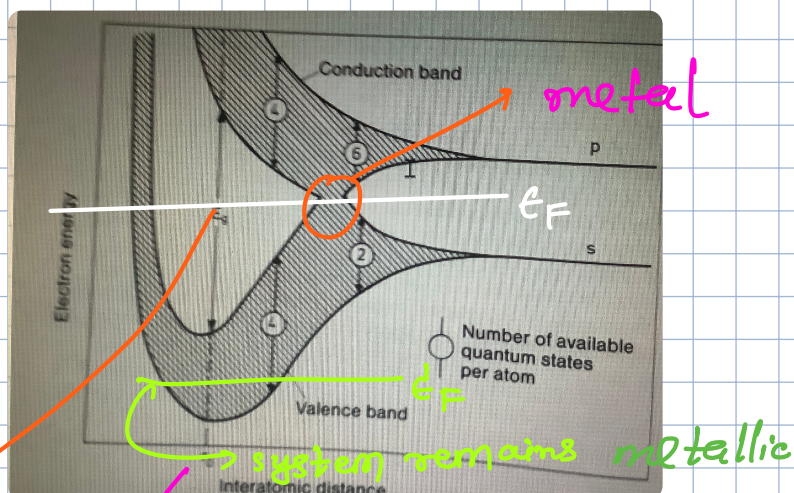


FIGURE 9.2. The bands arising from the 1s, 2s, and 2p atomic states of carbon (diamond) as a function of interatomic distance. [After W. Shockley, *Electrons and Holes in Semiconductors*, D. van Nostrand Co., Inc., New York (1950).]



semi conducting

eqm distance of solid

Goal:- we wanna understand how does one calculate these bonds are?

We'll use tight binding method.

not all encapsulating but captures a lot of the physics of it.

Two Approaches

① Free electron

$$KE \gg V$$

②

Assume solid has AOs & electrons are allowed to hop from one to other.

$$V \gg KE$$

Complementary pictures

We spend two lectures discussing this.

General symmetry properties

$$\nabla^2 \psi(\mathbf{r}) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E \psi(\mathbf{r})$$

$$V(\mathbf{r}) = V(\lambda_1, \lambda_2) ; \lambda_i = n_i a_i + n_2 a_2 + n_3 a_3$$

$$V(\mathbf{r}) = \sum_{\mathbf{q}} V_{\mathbf{q}} e^{i\mathbf{q} \cdot \mathbf{r}} \quad \left. \begin{array}{l} \text{kind of Fourier series} \\ \text{of the potential} \end{array} \right\}$$

$$\mathbf{q} = \hbar \mathbf{g}_1 + \hbar \mathbf{g}_2 + \hbar \mathbf{g}_3$$

$$\psi(\mathbf{r}) = \sum_{\mathbf{k}} C_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$\sum_{\mathbf{k}} \frac{\hbar^2 \mathbf{k}^2}{2m} C_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}} + \sum_{\mathbf{k}, \mathbf{q}} C_{\mathbf{k}} V_{\mathbf{q}} e^{i\mathbf{k} \cdot \mathbf{r}} e^{i\mathbf{q} \cdot \mathbf{r}} = E \sum_{\mathbf{k}} C_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$(\hbar^2 \mathbf{k}^2 / 2m - E) C_{\mathbf{k}} + \sum_{\mathbf{q}} V_{\mathbf{q}} C_{\mathbf{k}-\mathbf{q}} = 0$$

So essentially f.T. wavefn & we get

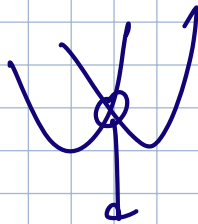
$$\left(\frac{\hbar^2 \mathbf{k}^2}{2m} - E \right) C_{\mathbf{k}} + \sum_{\mathbf{q}} V_{\mathbf{q}} C_{\mathbf{k}-\mathbf{q}} = 0$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{q}} C_{\mathbf{k}-\mathbf{q}} e^{i(\mathbf{k}-\mathbf{q}) \cdot \mathbf{r}} = e^{i\mathbf{k} \cdot \mathbf{r}} \left[\sum_{\mathbf{q}} C_{\mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{r}} \right]$$

Problem:- Calculating $V_{\mathbf{q}}$ is difficult

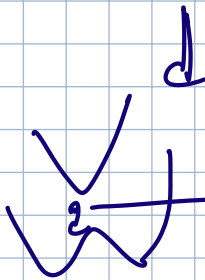
these are like the terms that couple different parabolas

$$U_{m\mathbf{k}}(\mathbf{r}) = U_{m\mathbf{k}}(\mathbf{r} + \mathbf{R})$$



hybridises states of diff $\mathbf{k}$

at crossing pts, we'll see a level repulsion.



$$\sim \text{Gap} \propto \sqrt{V}$$

extreme example

$$|\psi(x)|^2 = \left| \frac{e^{i\theta x} - e^{-i\theta x}}{2} \right|^2$$

$$|\psi(x)|^2 = \left| \frac{e^{i\theta x} + e^{-i\theta x}}{2} \right|^2$$

Another perspective
 $V_{\mathbf{q}}$ turns on backscattering in general.

free electron

potential

bonds

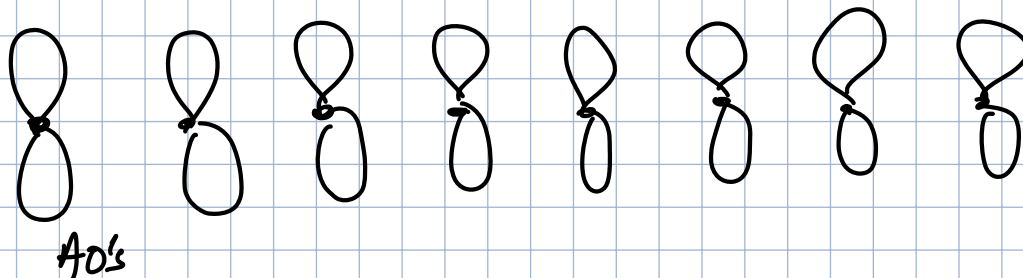
eff mass

Gaps

f. electron arises existence of semicond-

/insulators.

Another way :- Tight binding Approach.



SO $\psi_{\text{solid}} = \text{LCAO}$, for eg. $\psi = \sum_i \alpha_i \phi(r-r_i)$
 $\xrightarrow{\text{AO's}}$

$$H_A(r-r_n) \phi_i(r-r_m) = E_i \phi(r-r_m)$$

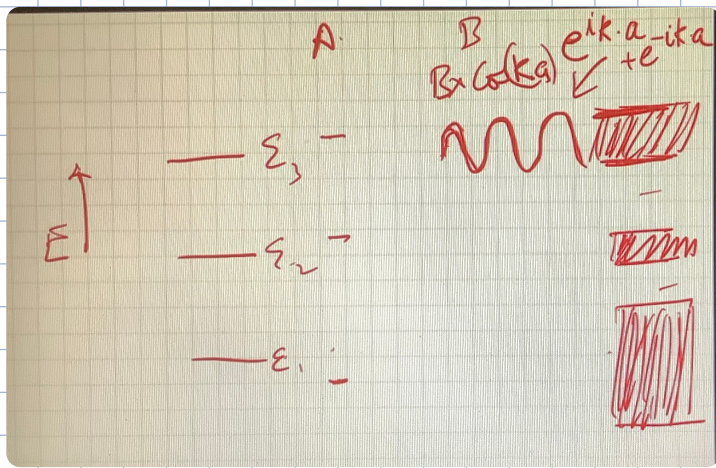
But now new $H = H_A + V$ $\xrightarrow{\text{existence of other sites around original}}$
 $= \underbrace{\frac{-\hbar^2}{2m} \nabla^2 + V_A(r-r_m)}_{\text{due to site } m} + \sum_{n \neq m} V_A(r-r_n)$

Variational approach

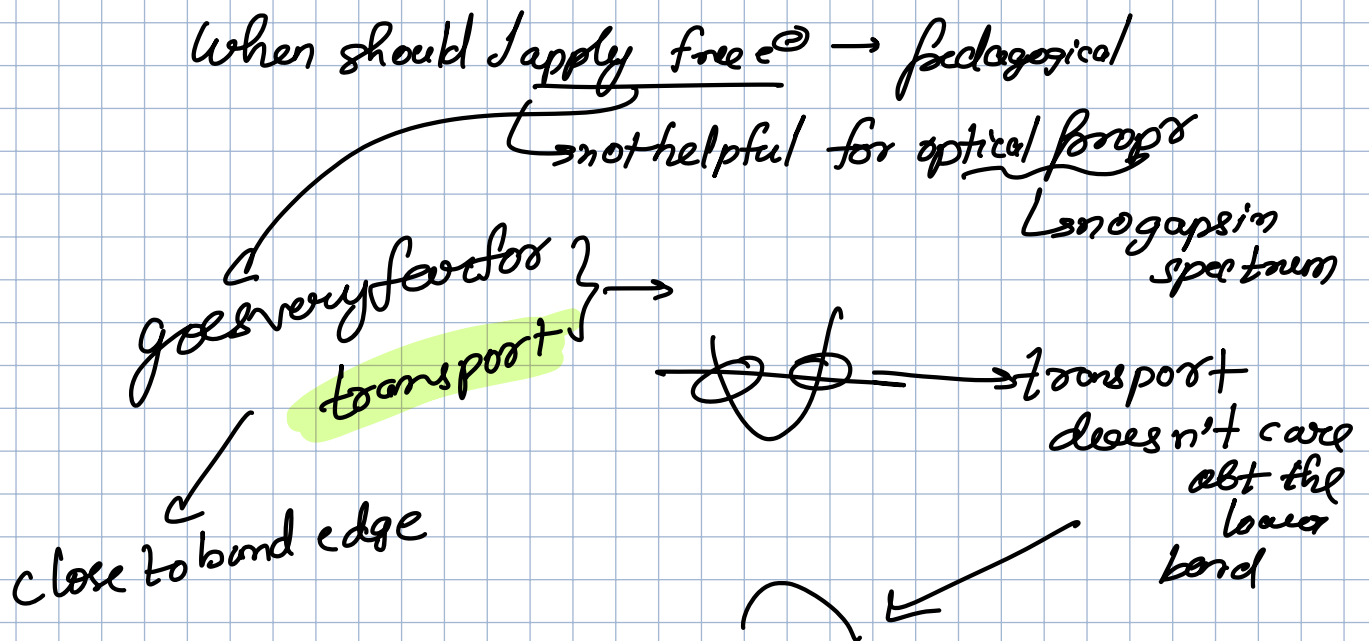
$\phi_{k \neq 0}$

$$\begin{aligned} \langle \phi_k | \phi_k \rangle &= \sum_{n,m} e^{ik(r_n - r_m)} \int \phi_k^*(r - r_m) \phi_k(r - r_n) dr \\ &\approx \sum_n \int \phi_k^*(r - r_n) \phi_k(r - r_n) dr = N \quad \text{(largest contribution when } r_n = r_m) \\ E(k) &\approx \frac{1}{N} \sum_{n,m} e^{ik(r_n - r_m)} \int \phi_k^*(r - r_m) [E_i + V(r - r_n)] \phi_k(r - r_n) dr \\ A &= - \int \phi_k^*(r - r_n) \nabla^2 \phi_k(r - r_n) dr \\ B &= - \int \phi_k^*(r - r_n) V(r - r_n) \phi_k(r - r_n) dr \\ E(k) &= E_i + \frac{A}{N} + \frac{B}{N} \end{aligned}$$

Maybe watch saddhoo's lectures



Book :-
E back & Luth.



But for optical props $\Rightarrow$ need TBM

free electron $\Rightarrow$

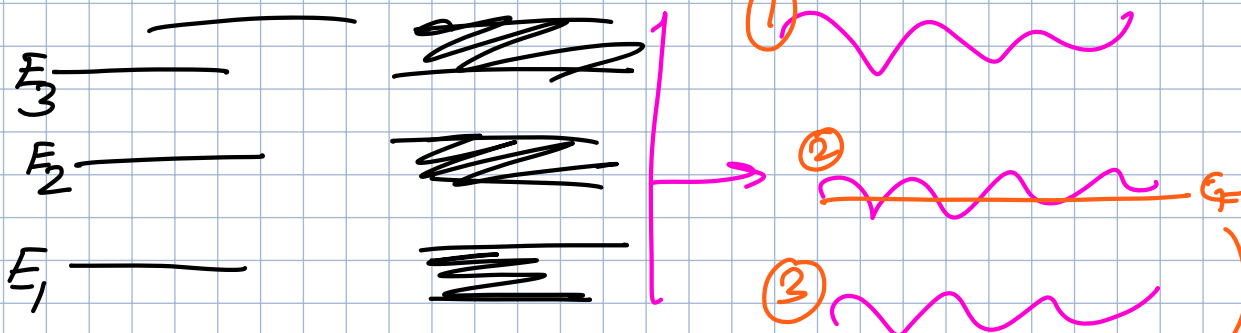
Arpes
shin light
look at e^- inside
core $\rightarrow$ 10 eV below Fermi energy
transport $\rightarrow$

Q.

When do we stop adding orbitals?

→ depends on the physics of what you want to explore.

for e.g. ① Transport



transport won't care "a lot" about bands ① & ③

But suppose we ARPES, which can give info on core orbitals, we have to take lower orbitals into account.